

Shot noise in diffusive ferromagnetic metals

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We show that shot noise in a diffusive ferromagnetic wire connected by tunnel contacts to two ferromagnetic electrodes can probe the intrinsic density of states and the extrinsic impurity scattering spin-polarization contributions in the polarization of the wire conductivity. The effect is more pronounced when the electrodes are perfectly polarized in opposite directions. While in this case the shot noise has a weak dependence on the impurity scattering polarization, it is strongly affected by the polarization of the density of states. For a finite spin-flip scattering rate the shot noise increases well above the normal state value and can reach the full Poissonian value when the density of states tends to be perfectly polarized. For the parallel configuration we find that the shot noise depends on the relative sign of the intrinsic and the extrinsic polarizations.

Shot noise is the low temperature temporal fluctuations of the electrical current through a conducting structure caused by the randomness of the electron scattering and the Fermi statistics. In the past years the shot noise has been extensively studied in different types of mesoscopic structures¹. It has been revealed that the shot noise measurements provide valuable information about the charge transport process which are not extractable from the mean conductance. For a fully random transmission of the electrons (e. g., through a tunnel contact) the current fluctuations $\Delta I(t)$ around the mean current \bar{I} is described by a Poissonian noise power $S = 2e\bar{I}^2$.

Correlations can reduce the shot noise below the Poissonian value. In a diffusive metal the restriction imposed by the Pauli exclusion principle on the random scattering of electrons from the impurities reduces the noise power by a universal factor of one-third^{3,4}. Coulomb repulsion introduces another source of the correlations which changes the shot noise in the diffusive metals^{4,5} as well as a nondegenerate electron gas⁶. Probing correlations and interactions by the shot noise measurement has been the subject of many investigations in the recent years^{1,7}. In contrary effects of spin-dependent correlations on the current fluctuations through a diffusive conductor has received little attention. In particular a natural question is that what happens to the one-third shot noise in a diffusive *ferromagnetic* metal in which additional correlations can be introduced between carriers of opposite spins by the interplay between the spin polarization of the density of states (DOS) of the Fermi level and the spin-dependent scatterings.

Here we propose that the shot noise in ferromagnetic metals can probe such correlations between spin up and down electrons. The basic idea is that the spin-dependent scattering including the spin-orbit coupling and the normal and the magnetic impurity scatterings are correlated with the imbalance in the Fermi level DOS of opposite spins. We show that such correlations change the current fluctuations in a way that shot noise acquires different dependence and sensitivity on the *intrinsic* spin-dependence due to the polarization of the conduction band, and the *extrinsic* spin polarization induced by the impurity scattering. Our finding is important in the con-

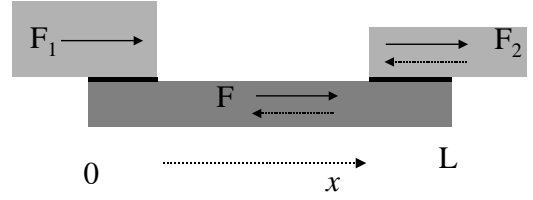


FIG. 1: Schematic of the studied spin-valve structure.

text of the anomalous Hall effect⁸. In ferromagnets the Hall resistance contains an anomalous term that is proportional not to the external field but to the magnetization of the ferromagnet. The question whether the anomalous Hall effect originates from the intrinsic⁹ spin-polarization of the band structure or has an extrinsic¹⁰ origin, such as scattering by normal or magnetic disorder, has been a subject of controversy¹¹. Thus the shot noise measurement in ferromagnets can be proposed to probe the intrinsic and the extrinsic anomalous Hall resistances.

Our motivation also comes from the importance of the noise in spintronics¹² devices in view of applications. Recently spin-polarized shot noise has been studied theoretically in normal metals connected by the ferromagnetic terminals with collinear^{13,14} and noncollinear¹⁵ magnetization directions. In Ref.¹⁴ a semiclassical Boltzmann-Langevin theory of the spin-polarized current fluctuations in diffusive normal metals was developed. It was found that in a multi-terminal spin-valve structure the shot noise and the cross correlations measured between currents of two different ferromagnetic terminals can deviate substantially from the unpolarized values, depending on the relative orientation of the magnetizations, the degree of spin-polarization of the terminals and the strength of the spin-flip scattering in the normal conductor.

In this paper we study fluctuations of the electrical current through a diffusive ferromagnetic wire. We consider a full ferromagnetic spin-valve system as shown in Fig. 1. A diffusive ferromagnetic wire is connected through the tunnel contacts to two ferromagnetic reservoirs. The reservoirs F_1 and F_2 are held at equilib-

rium at the voltages 0 and V respectively. Each of the F-terminals and the connecting wire has a magnetization which can be pointed in two different collinear directions. The polarization of the electronic DOS in the connected F metal causes a spin-polarized tunneling through the tunnel contact which is characterized by a spin-dependent conductance. For spin conserving tunneling the conductance of the i th ($i = 1, 2$) contact has the form $g_{i\alpha} = g_{Ti}(1 + \alpha p_N)(1 + \alpha p_i)/2$. Here g_{Ti} is the tunnel conductance in a fully normal system, p_i the polarization of the i th terminal, $p_N = \sum_{\alpha} \alpha N_{\alpha} / \sum_{\alpha} N_{\alpha}$ the DOS polarization with N_{α} being the spin α DOS in the F-wire ($\alpha = \pm 1$ denote the spin indices).

In the semiclassical regime the electronic transport across the F-wire can be explained within an extension of the Boltzmann-Langevin theory, which is developed in Refs.^{13,14} to study the spin-polarized shot noise in normal metals. We generalize this theory to a ferromagnetic metal by including the spin-dependence of the conductivity and the spin-flip diffusion length. Here we proceed with the corresponding diffusion equations for the fluctuating current density $j_{\alpha}(x, t, \varepsilon) = \bar{j}_{\alpha}(x, \varepsilon) + \delta j_{\alpha}(x, t, \varepsilon)$ and the distribution function $f_{\alpha}(x, t, \varepsilon) = \bar{f}_{\alpha}(x, \varepsilon) + \delta f_{\alpha}(x, t, \varepsilon)$ of spin α electrons at the energy ε , which have the form

$$j_{\alpha} = -\sigma_{\alpha} \frac{\partial}{\partial x} f_{\alpha} + j_{\alpha}^c, \quad (1)$$

$$\frac{\partial}{\partial x} j_{\alpha} = -\frac{\sigma_{\alpha}}{2\ell_{sf\alpha}^2} (f_{\alpha} - f_{-\alpha}) + i_{\alpha}^{sf}, \quad (2)$$

where the spin-dependent conductivity in the F-wire $\sigma_{\alpha} = e^2 N_{\alpha} D_{\alpha}$, in which $D_{\alpha} = v_{F\alpha}^2 \tau_{\alpha} / 3$ is the spin α diffusion constant. The spin-dependent relaxation time τ_{α} is expressed in terms of the normal impurity τ_{α}^{imp} and the spin-flip scattering τ_{α}^{sf} relaxation times as $1/\tau_{\alpha} = 1/\tau_{\alpha}^{imp} + 1/2\tau_{\alpha}^{sf}$. The spin-flip diffusion length ℓ_{sf} is expressed as $1/\ell_{sf}^2 = 1/2\ell_{sf+}^2 + 1/2\ell_{sf-}^2$ with $\ell_{sf\alpha} = (D_{\alpha} \tau_{\alpha}^{sf})^{1/2}$. In the diffusive limit of $\ell_{imp} \ll L$ we will consider the more realistic case where ℓ_{sf} is much larger than ℓ_{imp} , but arbitrary compared to L .

In Eq. (1) j_{α}^c is the Langevin source of the current density fluctuations caused by the stochastic nature of the normal and the spin-flip scattering events. The spin is not conserved by the spin-flip scattering which leads to the appearance of an additional fluctuating divergent term i_{α}^{sf} in Eq. (2). We obtain the following results for the correlations of the fluctuating terms in Eqs. (1) and (2),

$$\begin{aligned} & \langle j_{\alpha}^c(x, t, \varepsilon) j_{\alpha'}^c(x', t', \varepsilon') \rangle = \\ & \delta_{\alpha\alpha'} \Delta \sigma_{\alpha} \left[\frac{2\tau_{\alpha}}{\tau_{\alpha}^{imp}} \Pi_{\alpha\alpha} + \frac{\tau_{\alpha}}{2\tau_{\alpha}^{sf}} (\Pi_{\alpha-\alpha} + \Pi_{-\alpha\alpha}) \right], \end{aligned} \quad (3)$$

$$\begin{aligned} & \langle i_{\alpha}^{sf}(x, t, \varepsilon) i_{\alpha'}^{sf}(x', t', \varepsilon') \rangle = \\ & (\delta_{\alpha\alpha'} - \delta_{-\alpha\alpha'}) \Delta \frac{\sigma_{\alpha}}{2\ell_{sf\alpha}^2} [\Pi_{\alpha-\alpha} + \Pi_{-\alpha\alpha}], \end{aligned} \quad (4)$$

in which $\Delta = \delta(x - x')\delta(t - t')\delta(\varepsilon - \varepsilon')$ and $\Pi_{\alpha\alpha'} = \bar{f}_{\alpha}(x, \varepsilon)(1 - \bar{f}_{\alpha'}(x', \varepsilon'))$. Eqs. (3-4) are the extension

of the corresponding relations obtained for the normal transport^{1,3}. Thus we find that the current correlations are determined by the mean distribution of the spin α electrons.

Equations for the mean distribution functions are deduced by combining Eqs. (1) and (2) as

$$\frac{\partial^2}{\partial x^2} \bar{f}_{\alpha} = \frac{1}{2\ell_{sf\alpha}^2} (\bar{f}_{\alpha} - \bar{f}_{-\alpha}), \quad (5)$$

which for the two terminal structure of Fig. (1) has solutions of the form,

$$\begin{aligned} \bar{f}_{\alpha} = & f_1 + (f_2 - f_1) \left[a + b \frac{x}{L} \right. \\ & \left. + (\alpha - p_{\sigma}) \left(c \sinh \frac{\lambda x}{L} + d \cosh \frac{\lambda x}{L} \right) \right]. \end{aligned} \quad (6)$$

Here $\lambda = L/\ell_{sf}$ measures the strength of the spin-flip scattering which can be expressed in terms of the normal state spin-flip strength λ_0 and the DOS polarization p_N as $\lambda = \lambda_0(1 - p_N^2 + \gamma(1 + p_N^2))^{1/2}$; $f_i = f_{FD}(\varepsilon - eV_i)$ is the Fermi-Dirac distribution function in the electrodes held at the voltages V_i and $p_{\sigma} = \sum_{\alpha} \alpha \sigma_{\alpha} / \sum_{\alpha} \sigma_{\alpha}$ stands for the polarization of the conductivity. Both the polarization of DOS as well as the spin dependent scattering rate contribute to p_{σ} . Disregarding the Fermi velocity polarization it can be written as

$$p_{\sigma} = \frac{(1 - p_N^2) p_W + 2\gamma p_N}{1 - p_N^2 + \gamma(1 + p_N^2)}, \quad (7)$$

Here $p_W = \sum_{\alpha} \alpha W_{\alpha}^{imp} / \sum_{\alpha} W_{\alpha}^{imp}$ is the polarization of the impurity scattering rate and $\gamma = 2W_0^{sf} / \sum_{\alpha} W_{\alpha}^{imp} \ll 1$ is the ratio between the spin-flip and the normal impurity scattering rates.

The unknown coefficients a, b, c, d in Eq. (6) are obtained by the boundary conditions. Assuming spin-conserving tunneling the boundary conditions are expressed as the conservation of the temporal spin α current through the contacts. The fluctuating spin α current in the contact i is written as

$$I_{i\alpha}(\varepsilon, t) = g_{i\alpha} [f_i - f_{\alpha}(x_i, t)] + \delta I_{i\alpha}, \quad (8)$$

where $\delta I_{i\alpha}$ are the intrinsic fluctuations of the current due to the scattering of the electrons by the tunnel barriers. The intrinsic current fluctuations have the correlations of a Poisson process of the form $\langle \delta I_{i\alpha} \delta I_{j\alpha'} \rangle = \delta_{\alpha\alpha'} \delta_{ij} e \bar{I}_{\alpha}$.

On the other hand we can calculate the spin α current at a given point x from the diffusion equations (1) and (2). The mean current is obtained from the distribution function, given by Eq. (6), via the relation

$$\bar{I}_{\alpha}(x, \varepsilon) = -g_{\alpha} \frac{\partial}{\partial x} \bar{f}_{\alpha}(x, \varepsilon). \quad (9)$$

in which $g_{\alpha} = \sigma_{\alpha} A / L$ are the spin dependent conductances of the F wire. For the fluctuations of the currents

at the points $0, L$ we obtain the results

$$\begin{aligned} \Delta I_\alpha(0, L) &= g_\alpha \sum_{\alpha'} q_{\sigma\alpha'} (\delta f_{\alpha'}(L, 0) - \delta f_{\alpha'}(0, L)) \\ &+ \alpha g_{-\alpha} s(\lambda) q_{\sigma\alpha} \sum_{\alpha'} \alpha' (\delta f_{\alpha'}(L, 0) - \cosh \lambda \delta f_{\alpha'}(0, L)) \\ &+ q_{\sigma\alpha} \delta \mathcal{I}_c^c(0, L) + \frac{\alpha}{2} \delta \mathcal{I}_s^c(0, L) + \left(\frac{1 - 2q_{\sigma\alpha}}{2} \right) \delta \mathcal{I}_{cs}^c(0, L), \end{aligned} \quad (10)$$

Here $q_{\sigma\alpha} = (1 + \alpha p_\sigma)/2$, $s(\lambda) = \lambda / \sinh \lambda$ and the Langevin fluctuating currents $\delta \mathcal{I}_c^c$, $\delta \mathcal{I}_s^c$ and $\delta \mathcal{I}_{cs}^c$ are given by

$$\delta \mathcal{I}_c^c(0, L) = A \sum_\alpha \int dx (i_\alpha^{\text{sf}} + j_\alpha^c \frac{\partial}{\partial x}) \phi_{c0,L}, \quad (11)$$

$$\delta \mathcal{I}_s^c(0, L) = A \sum_\alpha \alpha \int dx (i_\alpha^{\text{sf}} + j_\alpha^c \frac{\partial}{\partial x}) \phi_{s0,L}, \quad (12)$$

$$\delta \mathcal{I}_{cs}^c(0, L) = A \sum_\alpha \int dx (i_\alpha^{\text{sf}} + j_\alpha^c \frac{\partial}{\partial x}) \phi_{s0,L}, \quad (13)$$

with $\phi_{c0}(x) = 1 - x/L$, $\phi_{s0}(x) = \sinh[\lambda(1 - x/L)] / \sinh \lambda$, $\phi_{cL}(x) = x/L$, and $\phi_{sL}(x) = \sinh[\lambda x/L] / \sinh \lambda$.

Now we impose the current conservation rule at the contacts using the expressions for the spin α currents given by Eqs. (8-10), from which we obtain the coefficients a, b, c, d and the fluctuations of the spin α distributions $\delta f_\alpha(0, L)$ at the contact points $0, L$. The distribution fluctuations are expressed in terms of $\delta I_{i\alpha}$, and the Langevin currents $\delta \mathcal{I}_{c(s)}^c(0, L)$, and $\delta \mathcal{I}_{cs}^c(0, L)$. These results together with the relations for the correlations of the Langevin currents which can be calculated using Eqs. (11-13) and (3-4), allows us to calculate the correlation of the spin α electrons in each of the contacts. Thus the correlations of the charge current $S = \langle \Delta I \Delta I \rangle$ and the Fano factor $F = S / 2e\bar{I}$ are obtained. The expression for the Fano factor is too lengthy to be written down here. For simplicity we consider a symmetric double barrier structure for that $g_{T1} = g_{T2} = g_T$. In the absence of any polarization in the system our result reduces to Fano factor for a full normal metal double barrier structure¹

$$F_N = \frac{1}{3} \frac{12 + 12g + 6g^2 + g^3}{(2 + g)^3}, \quad (14)$$

where $g = g_T / g_N$ with g_N being the normal conductance of the wire. In the following we discuss the variation of the Fano factor with respect to F_N taking $g = 1$ and for $\gamma \ll 1$.

We consider perfectly polarized F-terminals, when the effect of spin-polarization induced by the terminals is most pronounced. The magnetization vectors in the terminals can have parallel and anti-parallel orientations. The anti-parallel (A) case presents a single configuration. For the parallel (P) case there are two configurations corresponding to the parallel (PP) and antiparallel

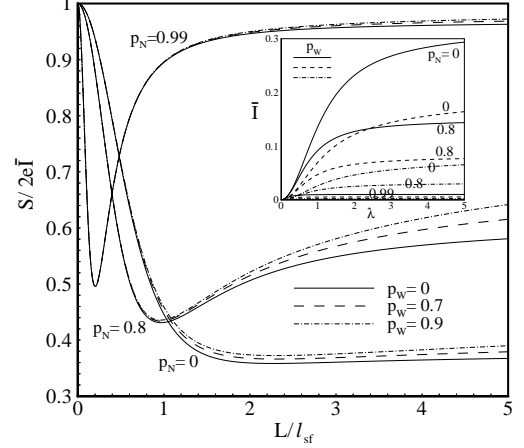


FIG. 2: Fano factor F as a function of the spin-flip scattering strength $\lambda = L/\ell_{\text{sf}}$ for the antiparallel magnetization alignment of the electrodes. We parameterize the magnetic properties of the wire with the DOS polarization p_N and the impurity scattering rate polarization p_W . Inset shows the corresponding mean current versus λ .

(PA) alignment of the polarization of the wire with respect to the magnetizations in the terminals.

The most striking result is obtained for the A configuration. In this case the perfectly antiparallel polarization of the terminals prevent charge transport through the wire in the absence of the spin-flip scattering. Thus one would expect more sensitive dependence of the fluctuating current on the strength of the spin-flip scattering which itself depends on the polarization of the wire. Fig. 2 shows the Fano factor F and the mean current \bar{I} (inset) dependence on λ for different p_N and p_W . The Fano factor is strongly modified with respect to the normal value given in Eq. (14). In the limit $\lambda \rightarrow 0$ the mean current I is vanishingly small and the Fano factor takes its full Poissonian value, 1. This does not depend on the values of p_N and p_W . The similar effect was found before for the normal spin-valve system^{13,14}. We can understand this behaviour by noting that for very small spin-flip strength a small current can flow through the wire by the electrons whose spins flip once. These few electrons travel almost uncorrelated through the wire resulting in a full Poissonian noise. For a finite λ the value of F strongly depends on the polarization. While for $p_N = p_W = 0$ the Fano factor decreases with λ and tends to the normal value Eq. (14) in the limit $\lambda \gg 1$, it has a nonmonotonic variation for finite p_N and p_W . The Fano factor at high λ increases above the normal value and reaches a value which is determined by the polarization. By approaching to the limit of perfect DOS polarization, $p_N \rightarrow 1$, the Fano factor tends to one. The nonmonotonic dependence on λ is the result of the interplay between the spin-flip process and the DOS polarization. While the former is dominant at $\lambda \ll 1$ and increases the mean number of

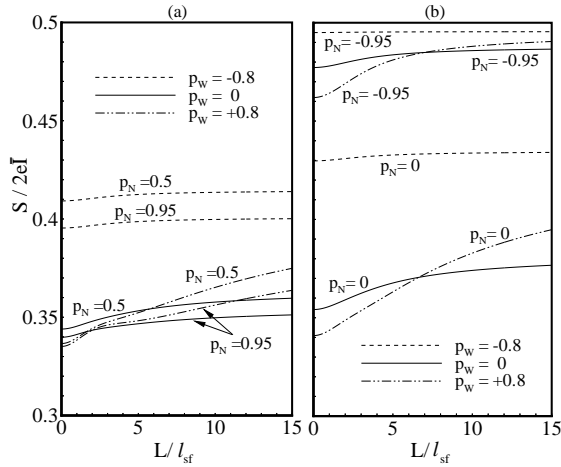


FIG. 3: Fano factor for parallel configuration of the electrodes magnetizations when the magnetization of the wire is parallel (a) and antiparallel (b) to the electrodes magnetization.

electrons whose spin flips once, the later imposes a restriction on the spin-flip current at higher (finite) λ s. Thus F develops a minimum at a λ which depends on p_N . It is interesting to note that in contrast to the mean current which can be modulated equivalently by varying p_N and p_W , the Fano factor F is mainly affected by the variation of p_N rather than p_W : The Fano factor for different p_N are well separated by varying slightly with p_W . Thus the shot noise measurement can be used to determine the intrinsic and the extrinsic contributions to the current polarization.

Let us now analyze the shot noise in the case of the P configurations. Figs. 3a and 3b illustrate the behavior of the shot noise in PP and PA configurations respectively. In the PP configuration the Fano factor has a weak vari-

ation with λ . But it varies considerably by increasing the polarization of the conductivity. Note that p_σ given in Eq. (7) is always close to p_W except when $p_N \rightarrow 1$ for which p_σ approaches one independently of p_W . The dominant dependence comes from the variation of p_W and the p_N -dependence is weak. In contrast to the case of the A configuration the shot noise changes with the relative sign of p_N and p_W for the P configurations. In the PA configuration for p_N approaching unity F become constant ($1/2$) independent of λ .

In conclusion we have presented a semiclassical Boltzmann-Langevin theory of shot noise in a fully ferromagnetic spin-valve consisting of a diffusive ferromagnetic wire connected by tunnel contacts to two perfectly polarized half-metallic ferromagnetic electrodes. We have shown that the shot noise can distinguish the intrinsic DOS polarization contribution from the extrinsic one induced by the scattering from the normal and the magnetic disorders. While the shot noise for an antiparallel configuration of the magnetization vectors in the electrodes has a weak dependence on the extrinsic impurity polarization, it is sensitive to the intrinsic DOS polarization. At a finite spin-flip scattering rate the shot noise has been shown to increase substantially above the unpolarized value and reaches the full Poissonian value for a perfectly polarized DOS. In the parallel configuration we have found that the shot noise is sensitive to the relative sign of the intrinsic and the extrinsic polarizations. Our result reveals importance of the shot noise as a possible probe to distinguish the intrinsic and the extrinsic anomalous Hall effects in ferromagnets.

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